Effects of Loading Rate and Pore Pressure on Compressive Strength of Rocks

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Outline

- Background and Rationale
- Objectives
- Rock Specimens and Preparation
- Laboratory Testing
- Test Results
- Conclusions and Discussions
Background and Rationale

Before

After

Dead weight (bridge)

Dead weight (bridge)

Dry zone

Dry zone

Saturated zone

Saturated zone

Water

Water
Background and Rationale…

Bhumipol Dam
http://roggerroll.wordpress.com

Srinakarin Dam
http://www.thai-tour.com
Masuda (2001) studies the effects of water on rock strength in granite and andesite.

The failure strength decreased linearly as the logarithm of the strain rate decreased.
Background and Rationale…

- Cobanoglu and Celik (2012) determine the uniaxial compressive strength tested in the dry and saturated conditions.

- The average saturated to dry strength ratios of travertines is 0.922.
Vasarhelyi (2003) determined the unconfined compressive strength of British sandstones.

Statistically the saturated UCS is 75.6% of the dry ($\text{UCS}_{\text{sat}} = 0.759\text{UCS}_{\text{dry}}$), while the saturated tangent and secant moduli are 76.1 and 79.0% of the dry samples respectively.
Li et al. (2012) study the influence of water content and anisotropy on the strength and deformability of sedimentary rocks.

The influence of water are reflected as a reduction of Young's modulus and increase of Poisson's ratio.
Objectives

- Determine the effects of pore pressure on the compressive strength and elastic properties of granite, marl and marble.

- Determine stress rate and confining pressure effect on the rock compressive strength.

- Assess the predictive capability of three-dimensional failure criteria that can be applied in the design and stability analysis of rock embankments and foundations under dry and saturated conditions.
Rock Specimens and Preparation

- Granite
- Marl
- Marble
The specimens are submerged under water in a pressure vacuum chamber.
Water Content

The water content \((W)\) in rock can be calculated by:

\[
W = \left( \frac{W_w}{W_s} \right) \times 100
\]  

(1)

where 

\(W_w\) = mass of water in rock 
\(W_s\) = dry mass of rock
Laboratory Testing...

Granite

\[ W_{\text{ave}} = 0.141 \pm 0.03 \]

Marble

\[ W_{\text{ave}} = 2.705 \pm 0.62 \]

Marl

\[ W_{\text{ave}} = 0.093 \pm 0.03 \]
Laboratory Testing…

A polyaxial load frame (Fuenkajorn & Kenkhunthod, 2010)
Laboratory Testing...

- Applied loading rate varies from 0.001, 0.01, 0.1, 1 and 10 MPa/s

- Applied confining pressure ($\sigma_3$) varies from 0, 3, 7, 12 MPa.
Laboratory Testing...

Dry condition

![Diagram of a cube with forces σ₁ and σ₃ in a dry condition.]

Saturated condition

![Diagram of a cube with forces σ₁ and σ₃ in a saturated condition.]

(Perforated Neoprene sheet)

(Neoprene sheet)
Laboratory Testing…

Hydraulic Cylindrical
Laboratory Testing...
Laboratory Testing…
Laboratory Testing…

- Axial stresses ($\sigma_1$) is increased until failure occurs.

- The axial strain, lateral strain, and time are monitored.
Rock Samples after Testing

\[ \sigma_3 = 0 \text{ MPa} \]

\[ \sigma_3 = 3 \text{ MPa} \]

\[ \sigma_3 = 7 \text{ MPa} \]

\[ \sigma_3 = 12 \text{ MPa} \]

\[ \dot{\sigma}_1 = 1 \text{ MPa/s} \]

\[ \dot{\sigma}_1 = 0.001 \text{ MPa/s} \]

Extension failure mode

Shear failure mode
Test Results

- **Strength of rock**
  - Maximum compressive strength

- **Coulomb criterion**
  - Shear strength, $\tau$
  - Cohesion, $c$
  - Internal friction angle, $\phi$

- **Elastic parameters**
  - Elastic modulus, $E$
  - Poisson’s ratio, $\nu$

- **Strain energy density criterion**
  - Distortional strain energy, $W_d$
  - Mean strain energy, $W_m$
Test Results...

- **Granite**
  - $\sigma_3 = 0$ MPa
  - $\sigma_{1,f}$ (MPa)
    - 0.0001
    - 0.001
    - 0.01
    - 0.1
    - 1
    - 10

- **Marl**
  - $\sigma_3 = 0$ MPa
  - $\sigma_{1,f}$ (MPa)
    - 0.0001
    - 0.001
    - 0.01
    - 0.1
    - 1
    - 10

- **Marble**
  - $\sigma_3 = 0$ MPa
  - $\sigma_{1,f}$ (MPa)
    - 0.0001
    - 0.001
    - 0.01
    - 0.1
    - 1
    - 10

- $\frac{\partial \sigma_1}{\partial t}$ (MPa/s)
  - 0
  - 50
  - 100
  - 150
  - 200
  - 250
  - 300
  - 3
  - 7
  - 12

- $\frac{\partial \sigma_3}{\partial t}$ (MPa/s)
  - 0
  - 50
  - 100
  - 150
  - 200
  - 250
  - 300
  - 3
  - 7
  - 12
Test Results…

- **Coulomb Criterion**

  The shear strength ($\tau$) can be represented by this equation

  $$\tau = c + \sigma_n \tan \phi$$  \hspace{1cm} (2)

  where $\sigma_n$ = the normal stress, 
  $c$ = the cohesion, 
  $\phi$ = friction angle.
Cohesion

\[ c = b \left( \frac{1 - \sin \phi}{2 - \cos \phi} \right) \] (3)

Internal friction angle

\[ \phi = \arcsin \left( \frac{m-1}{m+1} \right) \] (4)

\[ c = \chi \cdot \ln \left( \frac{\partial \sigma}{\partial t} \right) + \psi \] (5)

\[ \phi = \omega \cdot \ln \left( \frac{\partial \sigma}{\partial t} \right) + \iota \] (6)

The parameters \( \chi, \gamma, \omega, \iota \) are empirical parameters.
Substituting equations (5) and (6) into (2)

Equation (2) can be rewritten as

\[
\tau = \left[ \chi \cdot \ln(\partial \sigma \partial t) + \psi \right] + \sigma_n \tan \left[ \omega \cdot \ln(\partial \sigma \partial t) + \iota \right]
\] (7)
Test Results…

Marl

\[ c_{\text{Dry}} = 0.497 \ln(\frac{\sigma_1}{\dot{t}}) + 11.58 \text{ MPa} \]

\[ c_{\text{Sat}} = 0.479 \ln(\frac{\sigma_1}{\dot{t}}) + 11.48 \text{ MPa} \]

Marble

\[ c_{\text{Dry}} = 0.147 \ln(\frac{\sigma_1}{\dot{t}}) + 10.52 \text{ MPa} \]

\[ c_{\text{Sat}} = 0.170 \ln(\frac{\sigma_1}{\dot{t}}) + 10.58 \text{ MPa} \]

Granite

\[ c_{\text{Dry}} = 0.269 \ln(\frac{\sigma_1}{\dot{t}}) + 9.98 \text{ MPa} \]

\[ c_{\text{Sat}} = 0.290 \ln(\frac{\sigma_1}{\dot{t}}) + 9.55 \text{ MPa} \]
Test Results…

Granite

- $\phi_{\text{Dry}} = 0.431\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 61.4$ degrees
- $\phi_{\text{Sat}} = 0.364\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 59.6$ degrees

Marble

- $\phi_{\text{Dry}} = 0.431\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 61.4$ degrees
- $\phi_{\text{Sat}} = 0.364\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 59.6$ degrees

Marl

- $\phi_{\text{Dry}} = 0.263\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 43.8$ degrees
- $\phi_{\text{Sat}} = 0.166\ln\left(\frac{\sigma_1}{\dot{\tau}}\right) + 42.8$ degrees
Elastic Parameters

The elastic modulus (E), Poisson’s ratio (ν) can be determined by:

\[ G = \frac{1}{2} \left( \frac{\tau_{oct}}{\gamma_{oct}} \right) \]  \hspace{1cm} (8)

\[ \lambda = \frac{1}{3} \left[ \frac{3\sigma_m}{\Delta} - 2G \right] \]  \hspace{1cm} (9)

\[ E = 2G \left( 1 + \nu \right) \]  \hspace{1cm} (10)

\[ \nu = \lambda / \left( 2(\lambda + G) \right) \]  \hspace{1cm} (11)
Test Results…

The elastic parameters can be determined as a function of the loading rate as:

\[ E = \kappa \left( \frac{\partial \sigma_1}{\partial t} \right)^\xi \]  \hspace{1cm} (12)

\[ \nu = \alpha \ln \left( \frac{\partial \sigma_1}{\partial t} \right) + \beta \]  \hspace{1cm} (13)

The parameters \( \kappa, \xi, \alpha, \beta \) are empirical parameters.
Test Results...

Granite

\[ E_{\text{Dry}} = 12.546(\frac{\Delta \sigma}{\Delta t})^{0.094} \text{ GPa} \]
\[ E_{\text{Sat}} = 10.110(\frac{\Delta \sigma}{\Delta t})^{0.079} \text{ GPa} \]

Marl

\[ E_{\text{Dry}} = 9.9409(\frac{\Delta \sigma}{\Delta t})^{0.0858} \text{ GPa} \]
\[ E_{\text{Sat}} = 8.457(\frac{\Delta \sigma}{\Delta t})^{0.0755} \text{ GPa} \]

Marble

\[ E_{\text{Dry}} = 9.11(\frac{\Delta \sigma}{\Delta t})^{0.076} \text{ GPa} \]
\[ E_{\text{Sat}} = 7.14(\frac{\Delta \sigma}{\Delta t})^{0.076} \text{ GPa} \]
**Granite**

\[ \bar{v}_{\text{Dry}} = -0.0008 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.279 \]

\[ \bar{v}_{\text{Sat}} = -0.0008 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.282 \]

**Marl**

\[ \bar{v}_{\text{Dry}} = 0.0011 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.297 \]

\[ \bar{v}_{\text{Sat}} = 0.0001 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.292 \]

**Marble**

\[ \bar{v}_{\text{Dry}} = 0.001 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.280 \]

\[ \bar{v}_{\text{Sat}} = 0.003 \ln(\frac{\partial \sigma_1}{\partial t}) + 0.272 \]
Strain energy is the energy stored by a system undergoing deformation in 3D.

The strain energy density principle is applied here to describe the rock strength and deformation under different loading rates.
Test Results…

The distortional strain energy ($W_d$) at failure can be calculated as follows (Jaeger et al., 2007).

$$W_d = \frac{3}{4} \left( \frac{\tau^2_{oct,f}}{G} \right)$$  \hspace{1cm} (14)

The mean strain energy ($W_m$) at failure can be calculated as follows

$$W_m = \left( \frac{\sigma^2_m}{2K} \right)$$ \hspace{1cm} (15)
The elastic parameters $G$ and $K$ can be determined for each specimen using the following relations:

$$G = \frac{E}{2(1+\nu)}$$  \hspace{2cm} (16)

$$K = \frac{E}{3(1-(2\nu))}$$  \hspace{2cm} (17)

where $E = \text{Elastic modulus}$

$\nu = \text{Poisson’s ratio}$
Test Results…

The octahedral shear strength can be determined as:

\[
\tau_{\text{oct}} = \left[ \frac{1}{3} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) \right]^{1/2}
\] (18)

The mean stresses can be determined as:

\[
\sigma_m = \frac{1}{3} (\sigma_1 + 2 \sigma_3)
\] (19)
Granite

\[ W_{d,\text{Dry}} = 4.394W_{m,\text{Dry}} + 0.065 \]
\[ R^2 = 0.99 \]

\[ W_{d,\text{Sat}} = 3.923W_{m,\text{Sat}} + 0.106 \]
\[ R^2 = 0.99 \]

Marl

\[ W_{d,\text{Dry}} = 2.865W_{m,\text{Dry}} + 0.075 \]
\[ R^2 = 0.98 \]

\[ W_{d,\text{Sat}} = 2.501W_{m,\text{Sat}} + 0.094 \]
\[ R^2 = 0.99 \]

Marble

\[ W_{d,\text{Dry}} = 2.218W_{m,\text{Dry}} + 0.066 \]
\[ R^2 = 0.99 \]

\[ W_{d,\text{Sat}} = 2.018W_{m,\text{Sat}} + 0.087 \]
\[ R^2 = 0.95 \]
## Summary properties of rock

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<td>Mean strain energy, $W_m$</td>
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</tbody>
</table>
Conclusions and Discussion

- The compressive strength of dry specimens is higher than that of the saturated specimens due to effect of pore pressure.

- The strength of rock under low loading rate is lower than of high loading rate because under low loading rate rocks respond to stresses by ductile behavior not brittle behavior.
Conclusions and Discussion...

- The strength of the saturated specimens under high loading rates is reduced as the trapped pore water builds up the pore pressure.

- On the other hand, under low loading rates, the pore water has sufficient time to seep out from the specimens, making the rock behavior similar to dry condition.
Conclusions and Discussion...

- The elastic modulus of the dry specimens is higher than that of the saturated specimens that agrees with Li et al. 2012; Vasarhelyi, 2003.

- The strength criterion can be used to predict the strength and deformation of in-situ rocks under dry and saturated conditions.
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