# STABILITY PLANNING USING RELIABILTY TECHNIQUES

## ASEAN$^{++}$ 2013 Moving Forward

**November 11, 2013, Chiang Mai, THAILAND**

MAHIDOL UNIVERSITY

*Wisdom of the Land*


**Sanga Tangchawal, Ph.D.**

**Professor of Mining Engineering**

**Geoscience Program**

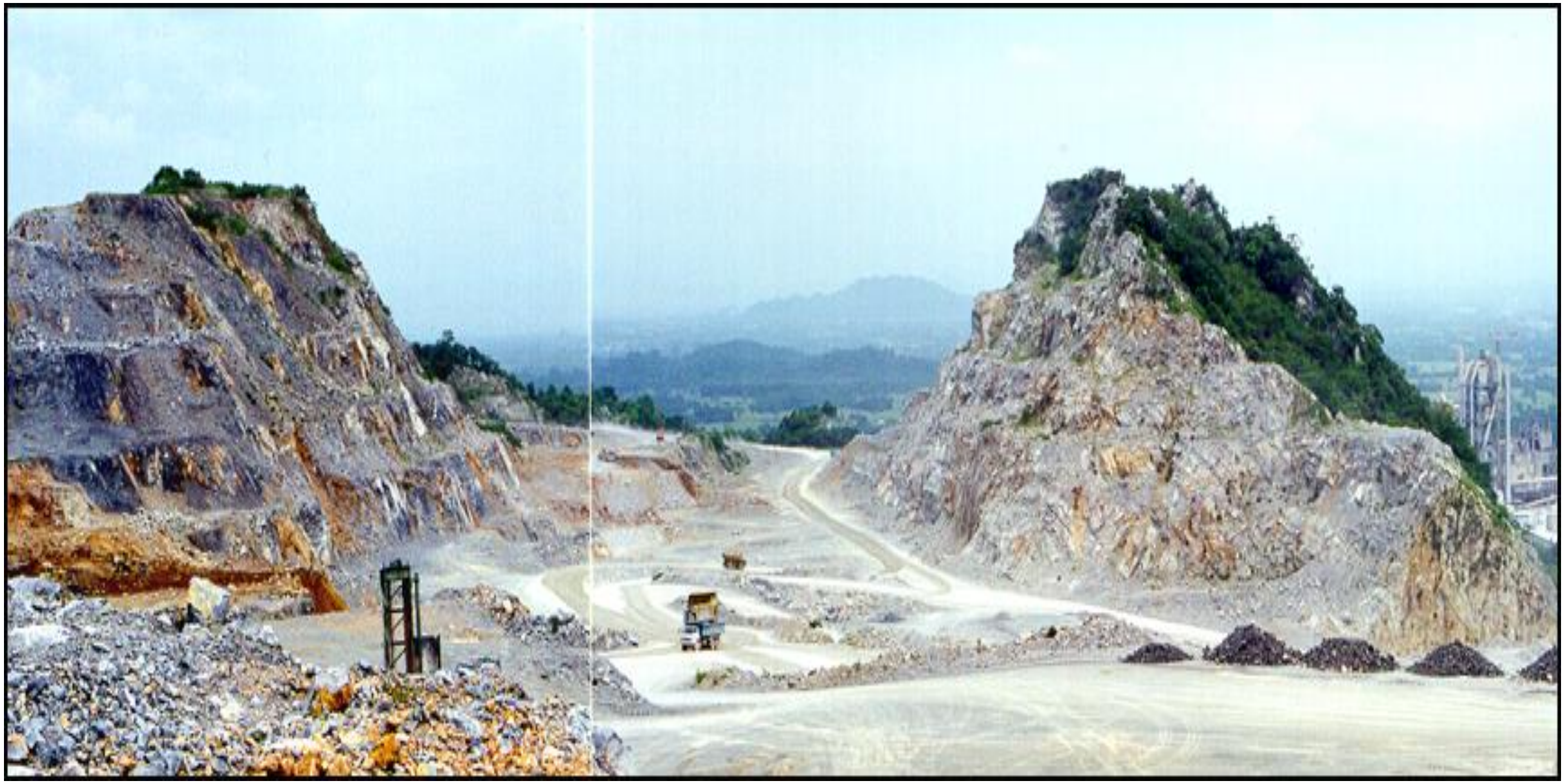
**Mahidol University, Kanchanaburi Campus,**

**Sai Yok, Kanchanaburi, THAILAND**

**Email: sanga.tang@gmail.com**

- **Stability Planning**
- **Based on Conventional Methods**

- **Large Limestone Quarry, Saraburi**

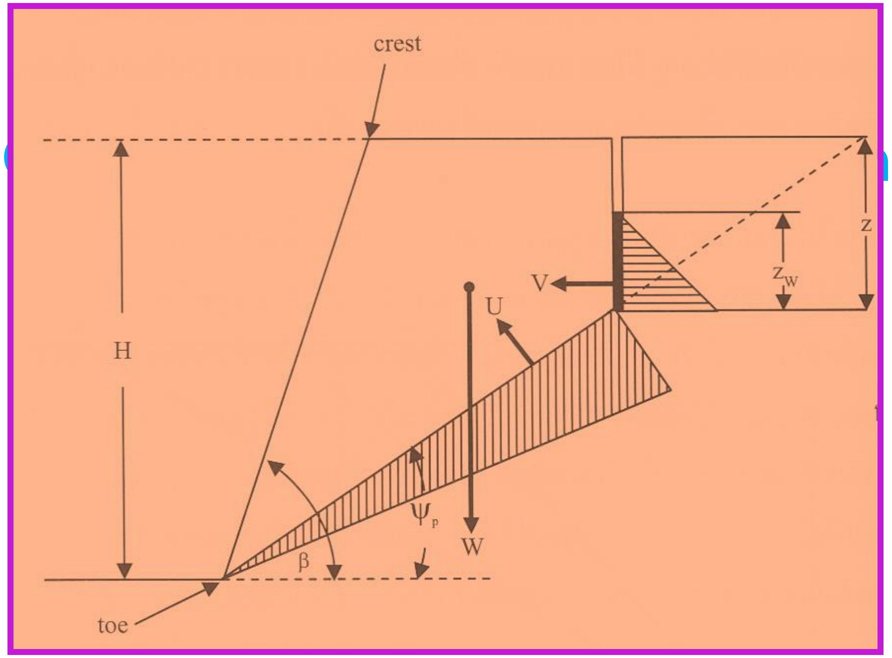


# Analysis on Plane Failure: Finite Slope (Bench Cut)

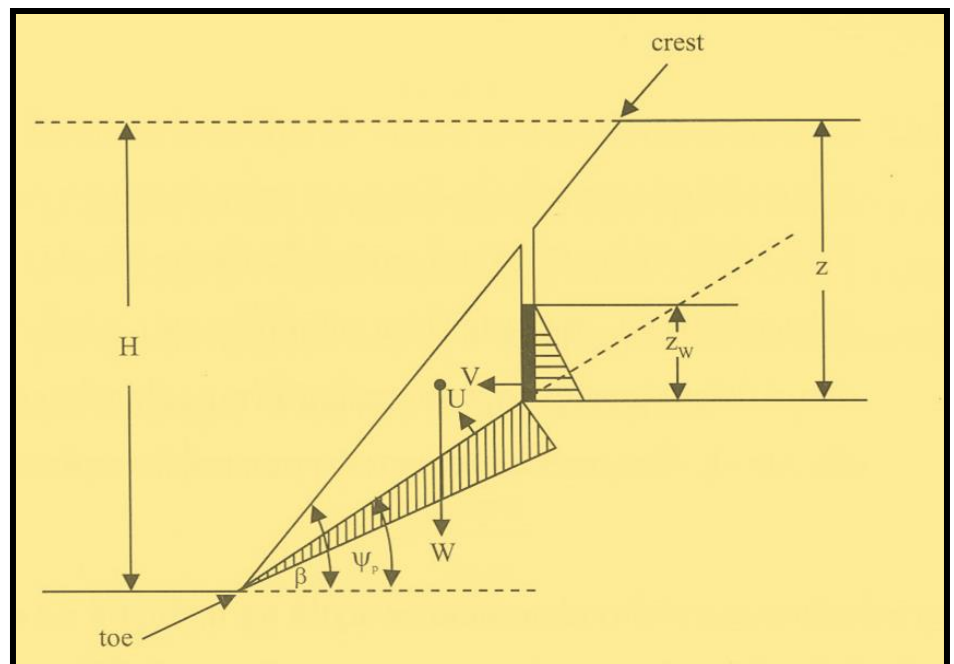
- **General Equation for F.S. calculation when having water forces involved of finite slope**

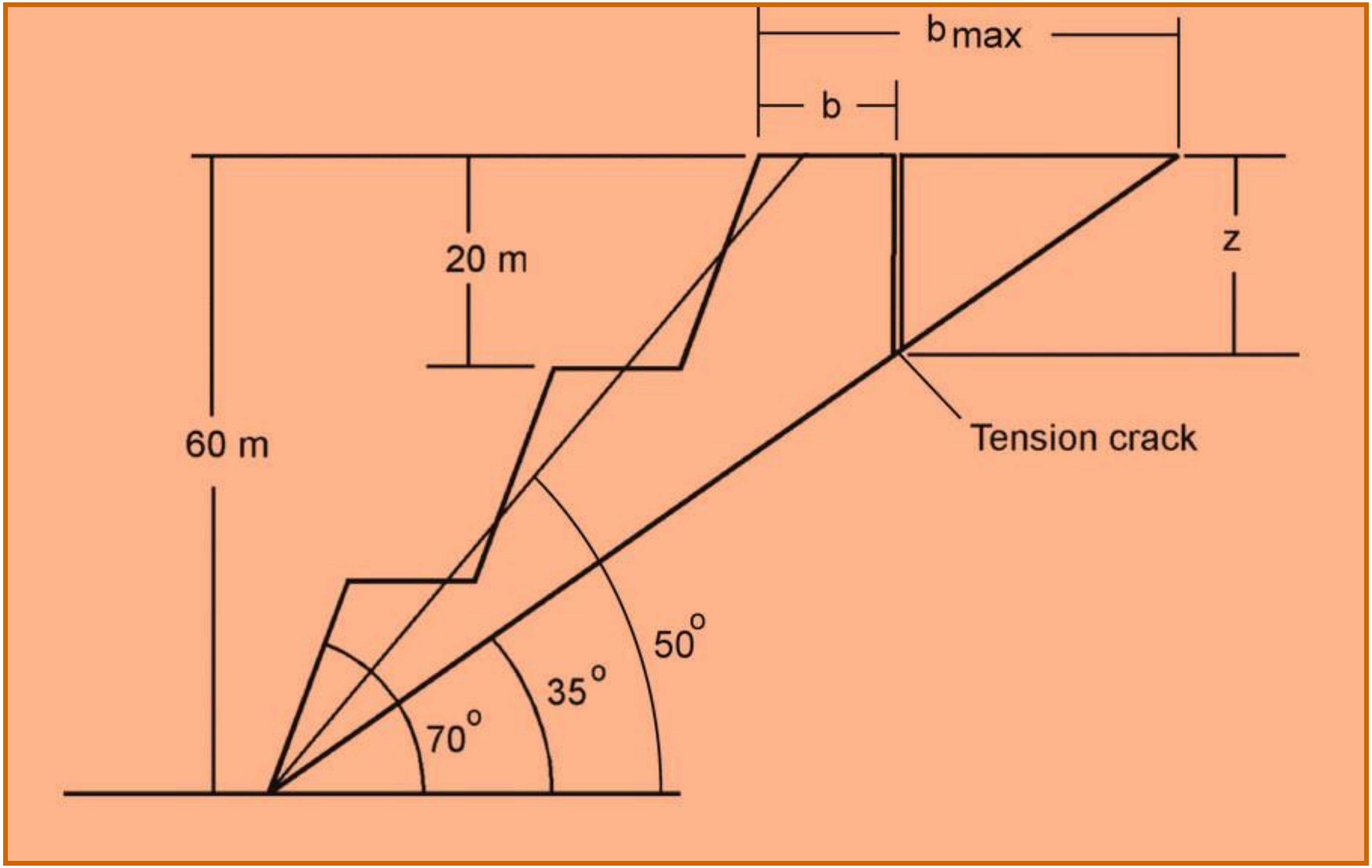
$$F.S. = \frac{(cL) + [(W\cos\psi_p) - U - (V\sin\psi_p)]\tan\phi}{(W\sin\psi_p) + (V\cos\psi_p)}$$

## Crack Occurs in Upper Bench

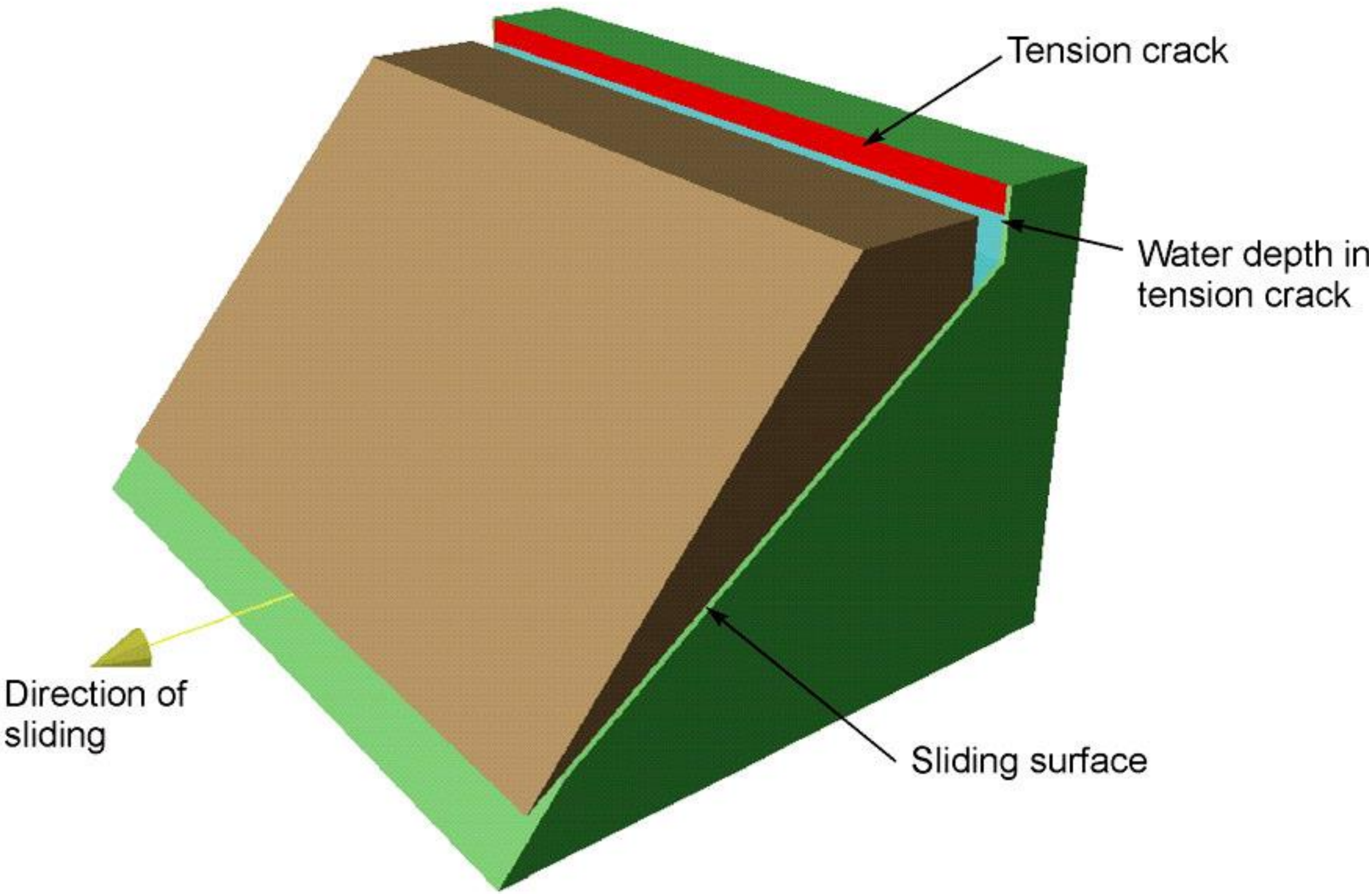



## Crack Occurs in slope face





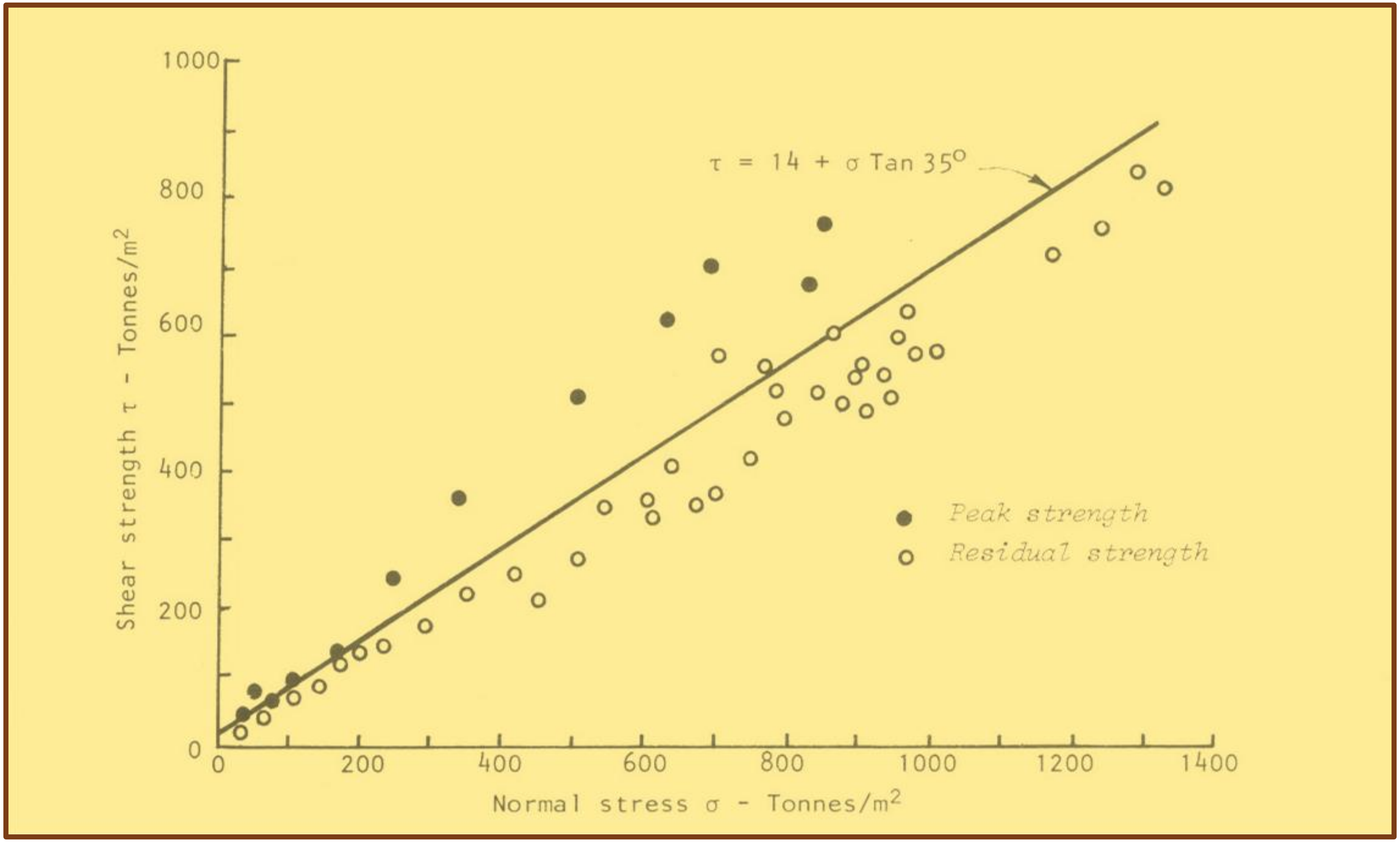

**Quarry Operation on various benches using overall angle slope as a basis in design with the possible failure plane.**


Tension crack
Water depth in
tension crack
Direction of
sliding
Sliding surface

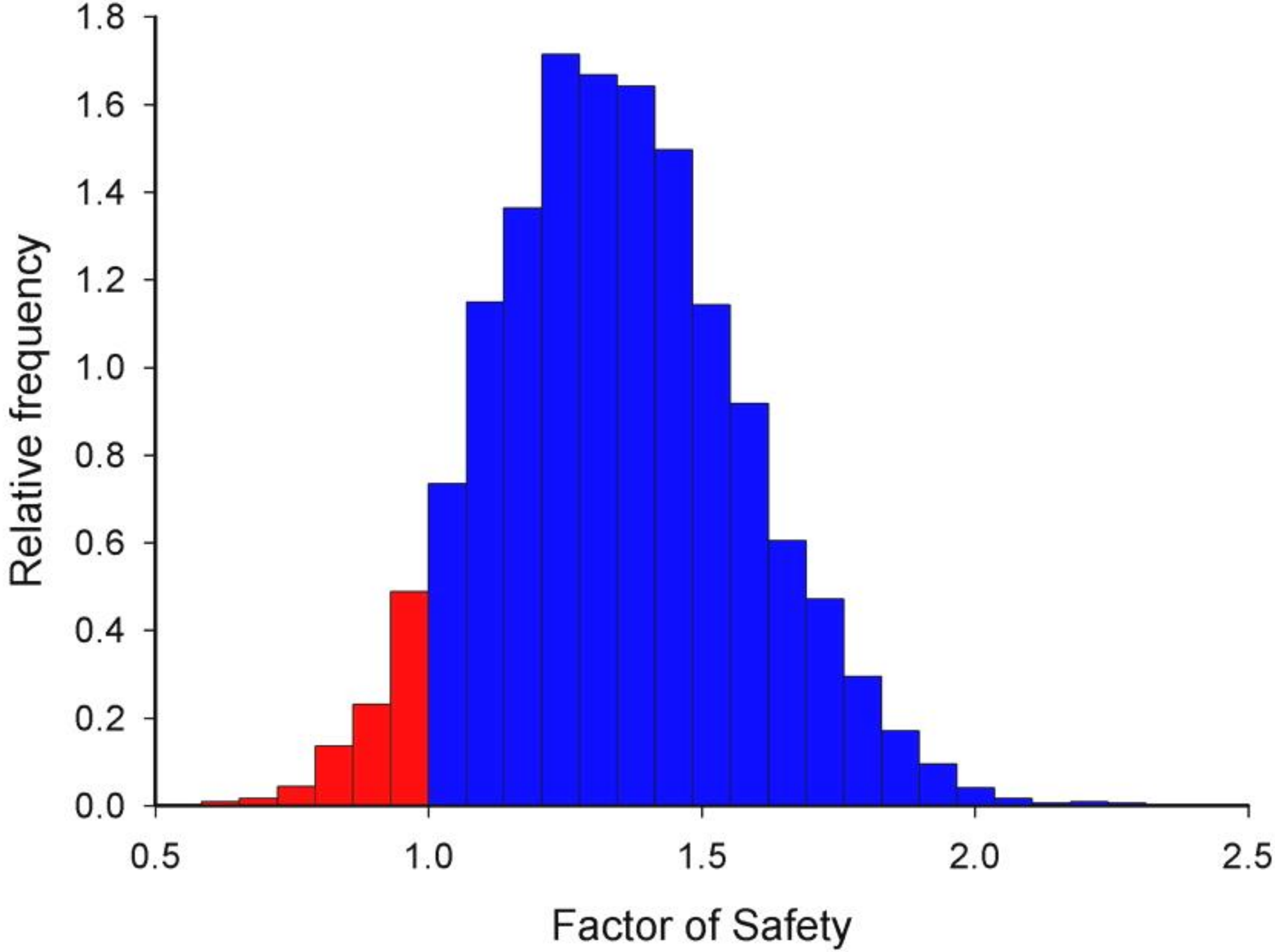
# A CASE HISTORY ON OVERALL SLOPE FAILURE

**Bench (Slope) Stability of Copper-Iron Porphyry Open Pit Mine, Southern Spain in 1969. (After Hoek and Bray, 1981).**

**Atalaya Open Pit Mine has the Problem of slope Stability on The Northern Side.**

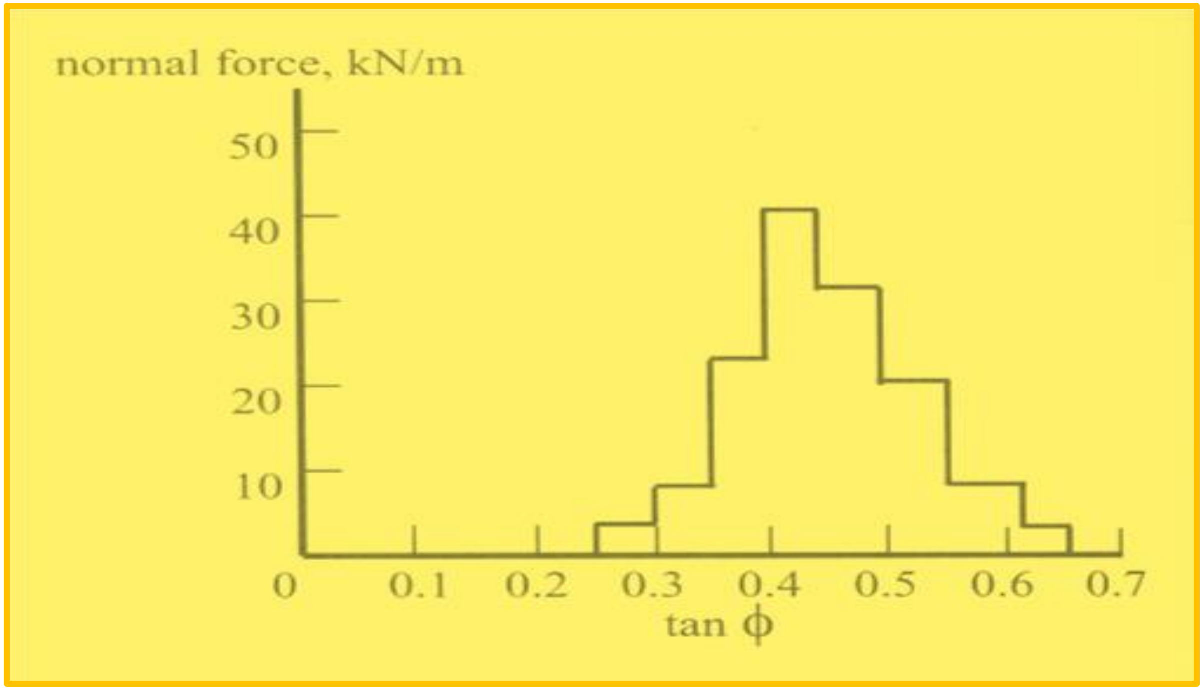



**Variation on shear strength values on Atlaya's open pit slope, southern Spain (After Hoek and Bray, 1981).**


Relative frequency
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
2.5
Factor of Safety

# Chance or Probability of Failure

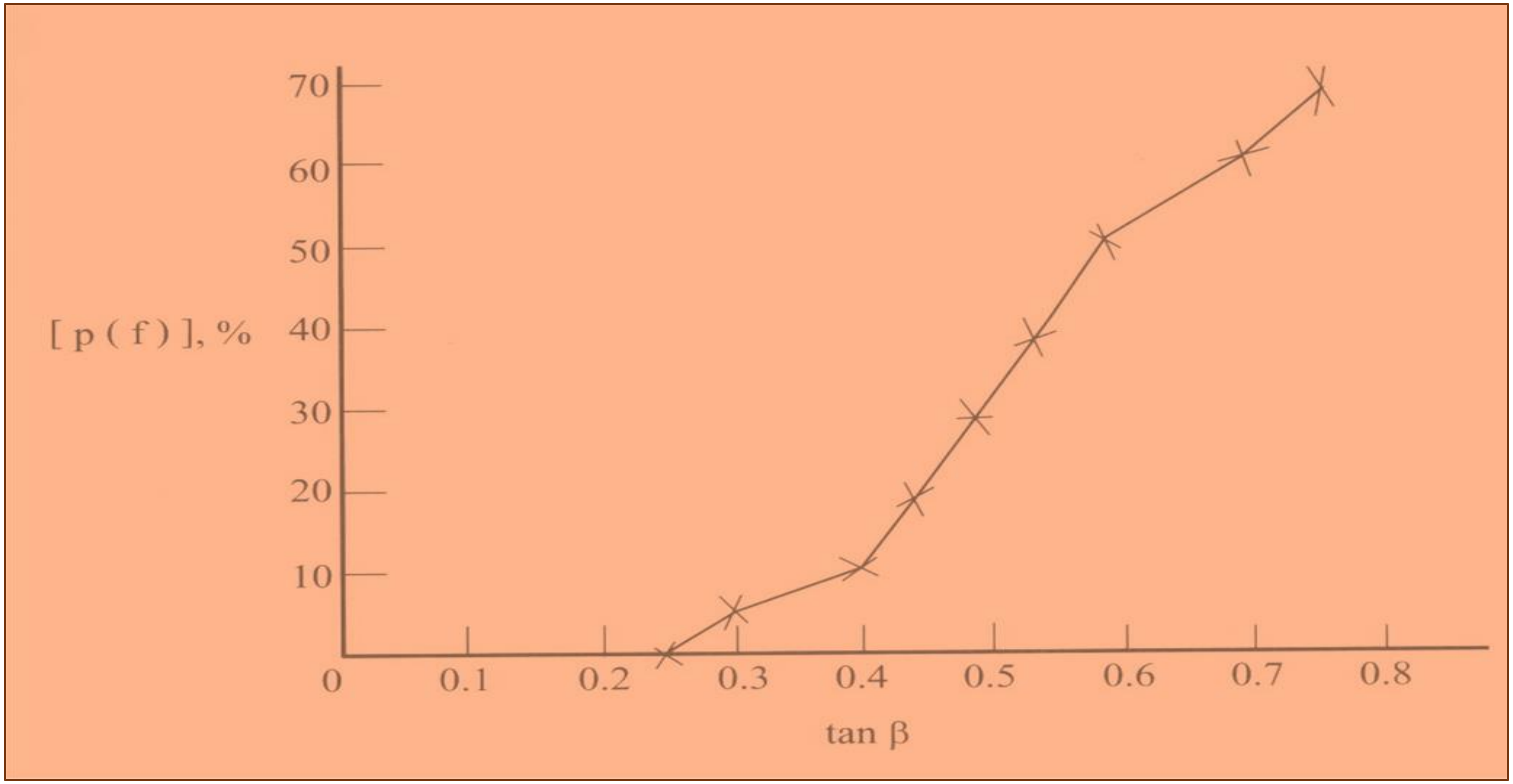
- Material properties such as cohesion, friction angle, unit weight have different values on the same construction site.




**Friction angle (ϕ) vs. Normal force on the failure plane.**

# Chance or Probability of Failure

- Distribution on property values and dimensions cause the uncertainty on the chance or probability of mass failure.



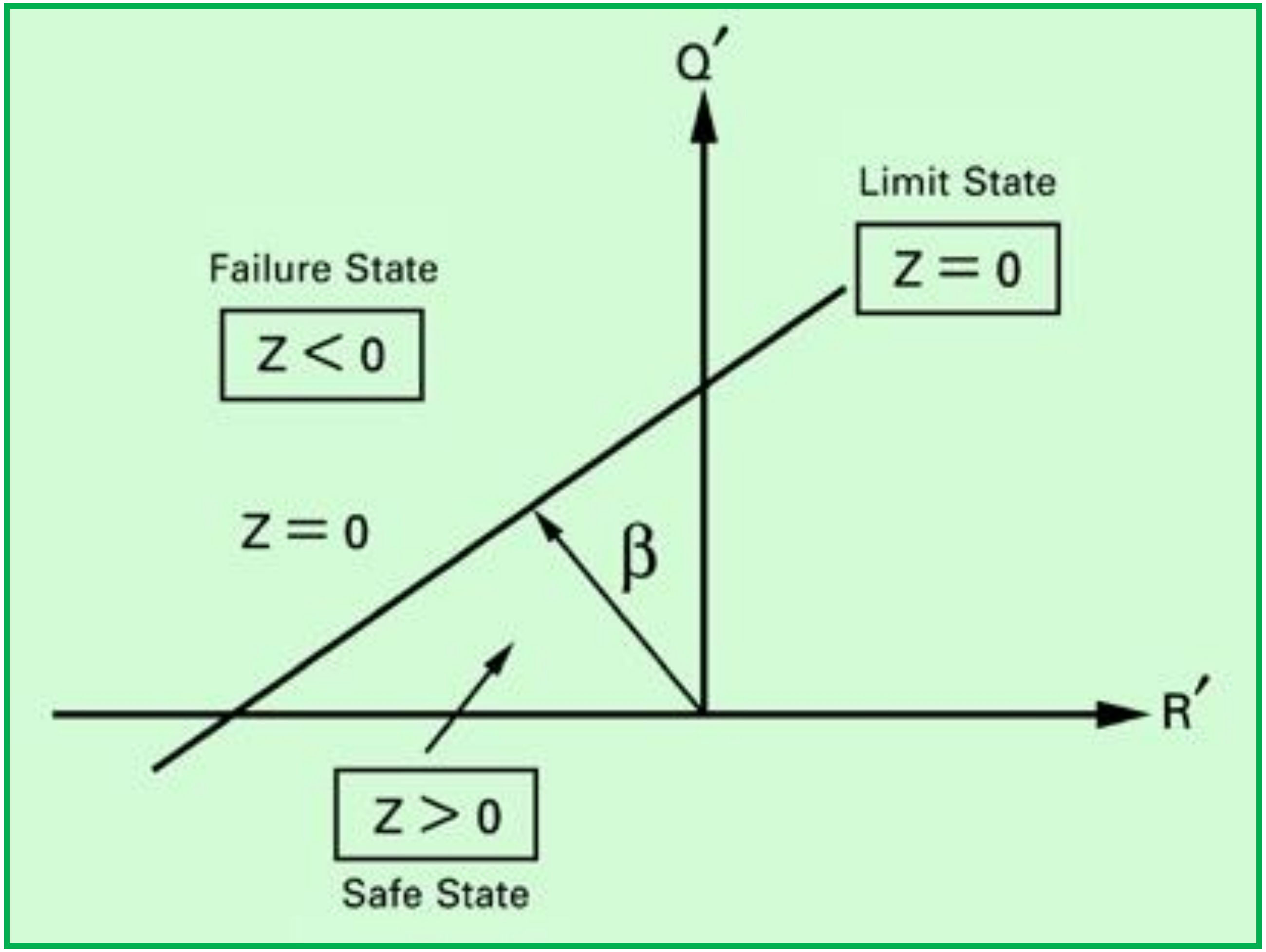

**Slope face angle (β) vs. Probability of failure [p (f)].**

# PROPOSED RISK MODELS (3 TYPES)

1. Based on the Safety Margin Concept

2. Based on the Probable Factor of Safety

3. Based on the Data Simulation (Trial Method)




**RISK MODEL I – Safety Margin (z) Concept: The limit, failure, safe state. Also the value of reliability index (β). Horizontal axis is the resistant (property) values (R′), vertical axis is the sliding (unstable) values (Q′). After Hasofer and Lind (1974).**

# RISK MODEL I: SAFETY MARGIN CONCEPT

$$Z = R - Q$$

$$(F.S.)_{mean} = \frac{R_{mean}}{Q_{mean}}$$

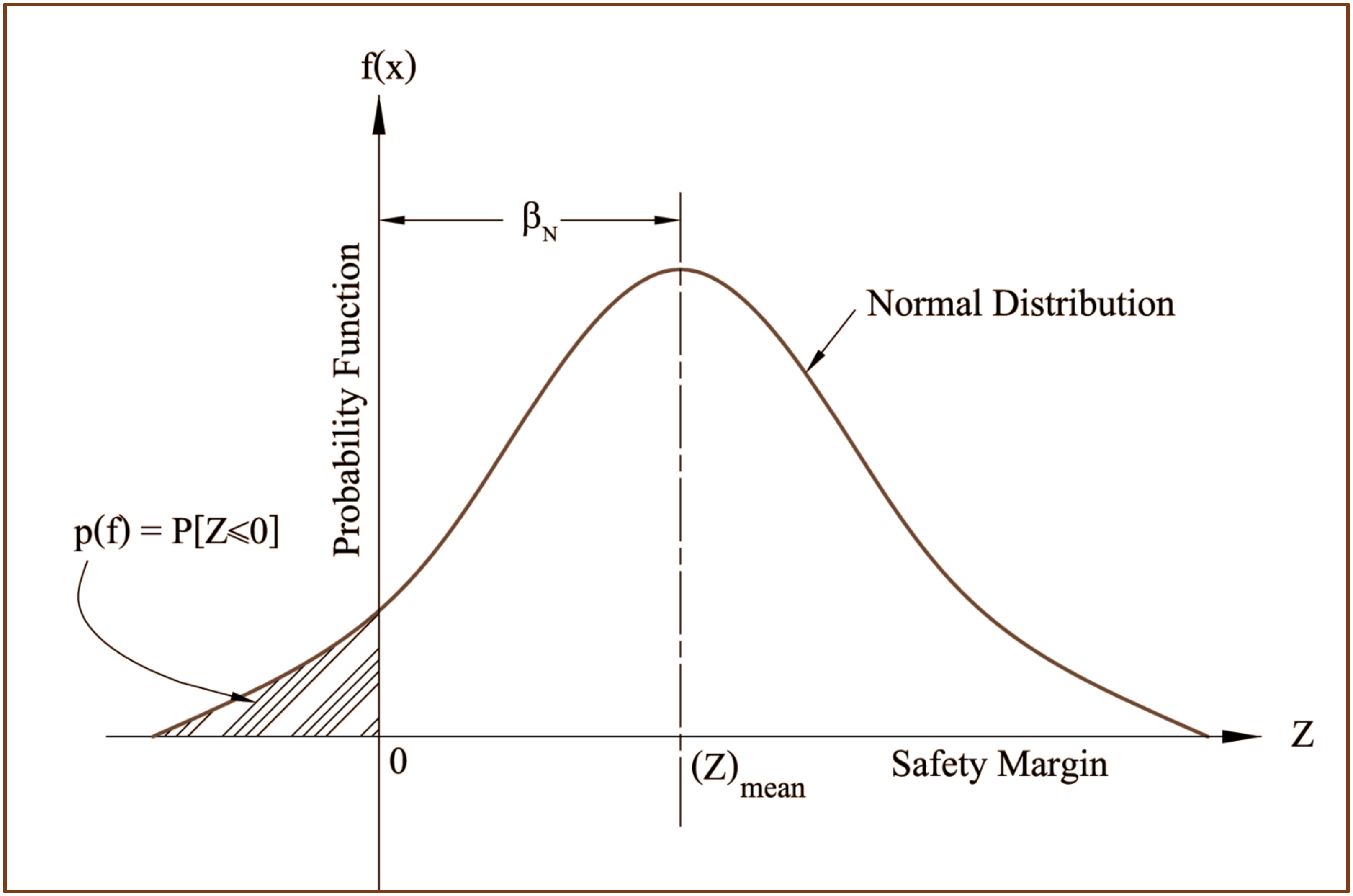
**Note: Z = Safety Margin, R = Resistant (Property) Values, Q = Sliding (Unstable) Values**

# RISK MODEL I: SAFETY MARGIN CONCEPT

$$\text{Reliability} = 1 - p(f)$$

$$p(f) = 1 - F(x) = 1 - F(\beta)$$

**Note: p (f) = Probability of Failure, F = Cumulative Distribution Function, β = Reliability Index**




**Assumption on data distribution on the 3 proposed risk models: normal and lognormal distribution. For risk model I: the representation graph above is the normal distribution of data and its reliability index ($\beta_N$).**

# RISK MODEL I: SAFETY MARGIN CONCEPT

## Reliability Index For the Normal Distribution of Input Data of risk model I

$$\beta_N = \frac{(R_{mean} - Q_{mean})}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

**Note: The value $\beta_N$ is the reliability index value for normal distribution.**

# RISK MODEL I: SAFETY MARGIN CONCEPT

**Reliability Index For the Lognormal Distribution of Input Data of risk model I**

$$\beta_{LN} = \frac{\ell n\left[\frac{R_{mean}}{Q_{mean}} \cdot \sqrt{\frac{1+(C.O.V.)_Q^2}{\{1+(C.O.V.)_R^2\}}}\right]}{\sqrt{\ell n[(1+(C.O.V.)_R^2)\,(1+(C.O.V.)_Q^2)]}}$$

**Note: The value $\beta_{LN}$ is the reliability index value for lognormal distribution.**

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

- **The Value of Factor of Safety assumed to be not constant.**

- **The Deterministic F.S. (from conventional method) has been recalculated by adding and subtracting their mean value (of each random variable) for 1 S.D.**

- **The new F.S. value named " The Most Likely Value of Factor of Safety, $(F.S.)_{MLV}$**

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

- **The Most Likely Value of Factor of Safety, $(F.S.)_{MLV}$ is**

$$(F.S.)_{MLV} = \frac{(F_1^{+} + F_1^{-}) + (F_2^{+} + F_2^{-}) + \ldots + (F_N^{+} + F_N^{-})}{2(N)}$$

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

**The standard deviation value based on the probable F.S., $\sigma_{FS}$ is**

$$\sigma_{FS} = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \ldots + \left(\frac{\Delta F_N}{2}\right)^2}$$

**Value of $\Delta F$ is**

$$\Delta F_1 = \left| F_1^{+} - F_1^{-} \right|$$

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

**Then the coefficient of variation based on the probable F.S., $(C.O.V.)_{FS}$ is**

$$(C.O.V.)_{FS} = \frac{\sigma_{FS}}{(F.S.)_{MLV}}$$

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

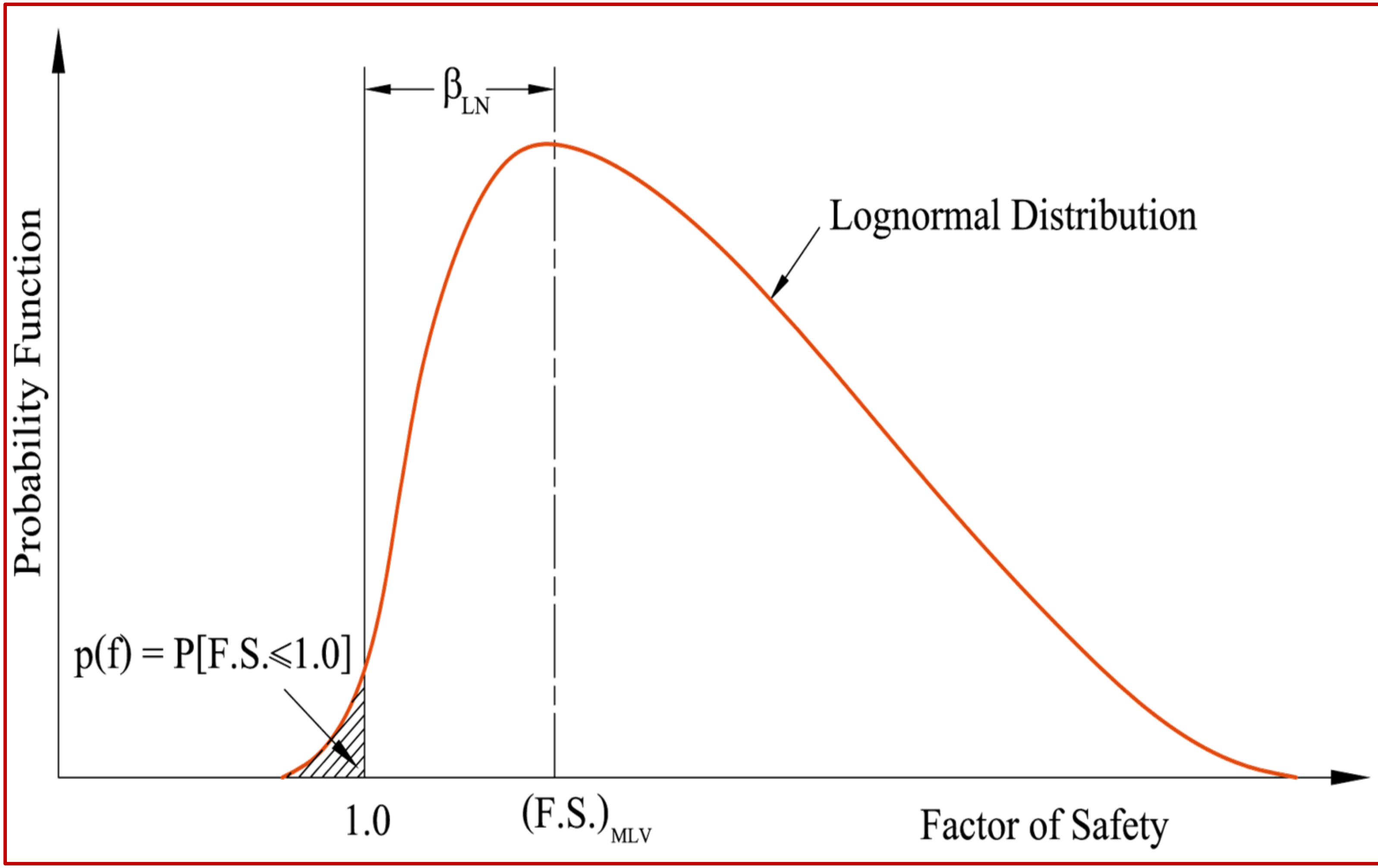
**Reliability Index For the Normal Distribution of Input Data of risk model II**

$$\beta_N = \frac{[(F.S.)_{MLV} - 1]}{\sigma_{FS}}$$

# RISK MODEL II: THE CONCEPT ON PROBABLE F.S.

## Reliability Index For the Lognormal Distribution of Input Data of Model II

$$\beta_{LN} = \frac{\ell n\left[\dfrac{(F.S.)_{MLV}}{\sqrt{1+(C.O.V.)_{FS}^{2}}}\right]}{\ell n[1+(C.O.V.)_{FS}^{2}]}$$




**Assumption on data distribution on the 3 proposed risk models: normal and lognormal distribution. For risk model II: the representation graph above is the lognormal distribution of data and its reliability index ($\beta_{LN}$).**

# RISK MODEL III: THE CONCEPT ON DATA SIMULATION ON F.S.

**Random Numbers (input data) between 0 and 1.0 are generated to be selected.**

**Sampling the probability density function values between – 4(SD) and + 4(SD) and calculated its F.S.**

# RISK MODEL III: THE CONCEPT ON DATA SIMULATION ON F.S.

- The F.S. value that is higher than 1.0 is accepted as the output.

- The F.S. value that is lower than 1.0 is being sent to repeat the process of generation, sampling, and recalculate.

# RISK MODEL III: THE CONCEPT ON DATA SIMULATION ON F.S.

- **Monte Carlo Technique on Simulation of (F.S.) values:**

$$p(f) = \frac{K_1}{(K_1 + K_2)}$$

$K_1$ = Number (F.S. < 1)

$K_2$ = Number (F.S. $\geq$ 1)

# RISK MODEL III: THE CONCEPT ON DATA SIMULATION ON F.S.

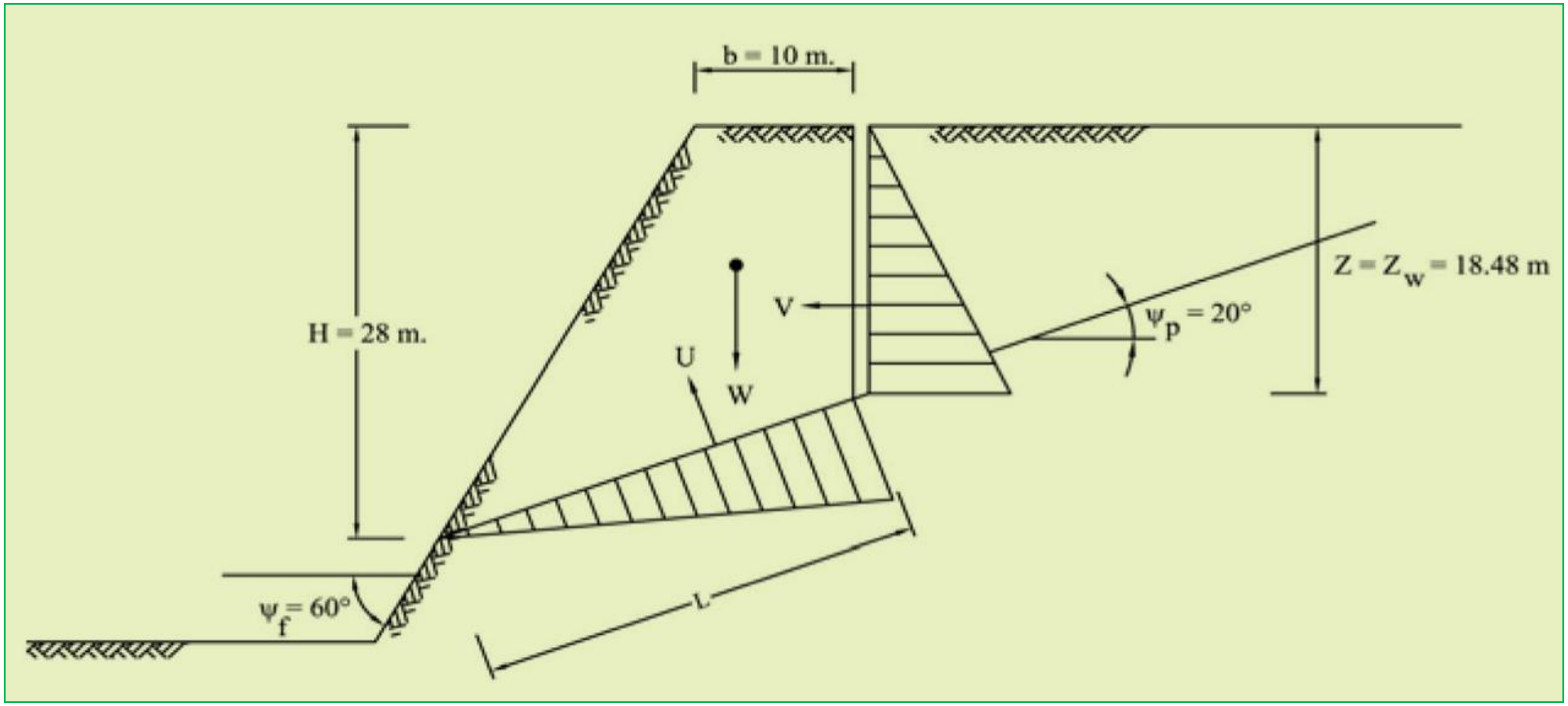
**The simulated value of p (f) for Model III is**

$$[p(f)]_{SIM} = \frac{(L - M)}{L}$$

**M is the number of F.S. > 1.0;**
**L is the repeat process number (100-10,000 times).**
**Simulation using the written program.**

**Note: The value of probability of failure is given in range not one single value.**

# Back Analysis: Slope Cut




**A plane failure on the quarry limestone. Tension crack occurs in slope face.** **All data value is in the next slide.**

# Back Analysis: Slope Cut

## Data given (single value) :

**H = slope height = 28 m, Slope face angle, $\theta = 60^O$, z = 18.48 m, Inclined angle of upper bench, $\Omega = 0^O$, $z_w/z = 1$**

## Data given (variation) :

| | Mean | S.D, | C.O.V. |
|---|---|---|---|
| Failure plane angle, $\psi_p$ | $20^O$ | $3^O$ | 15% |
| Cohesion | 100 kPa | 15 kPa | 15% |
| Friction angle, $\phi$ | $20^O$ | $3^O$ | 15% |
| Rock unit weight, $\gamma$ | 25.1 $kN/m^3$ | 3.765 $kN/m^3$ | 15% |

# Slope Cut (Normal Distribution)

## RESULT FROM MODEL I: SAFETY MARGIN

| Mean Factor of Safety, $(F.S.)_{mean}$ | Reliability Index, $\beta_N$ | Probability of Failure, p (f) |
|---|---|---|
| 1.02 | 0.1136 | 45.48% |

## RESULT FROM MODEL II: PROBABLE F.S.

| Most Likely Factor of Safety, $(F.S.)_{MLV}$ | Reliability Index, $\beta_N$ | Probability of Failure, p (f) |
|---|---|---|
| 1.02 | 0.1676 | 43.34% |

# Slope Cut (Lognormal Distribution)

## RESULT FROM MODEL I: SAFETY MARGIN

| Mean Factor of Safety, $(F.S.)_{mean}$ | Reliability Index, $\beta_{LN}$ | Probability of Failure, p (f) |
|---|---|---|
| 1.01 | 0.1517 | 50.03% |

## RESULT FROM MODEL II: PROBABLE F.S.

| Most Likely Factor of Safety, $(F.S.)_{MLV}$ | Reliability Index, $\beta_{LN}$ | Probability of Failure, p (f) |
|---|---|---|
| 1.01 | 0.1358 | 48.01% |

# Slope Cut

## RESULT FROM MODEL III: SIMULATION (Normal Data)

| Range of Factor of Safety | Reliability Index, $\beta_N$ | Probability of Failure, p (f) |
|---|---|---|
| 0.66 –1.45 | 0.1840 | 42.70 % |

## RESULT FROM MODEL III: SIMULATION (Lognormal Data)

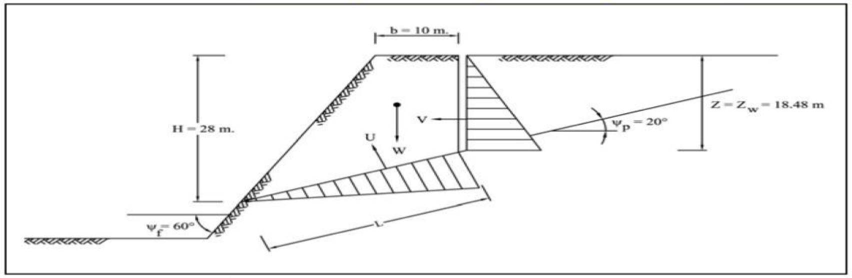
| Range of Factor of Safety | Reliability Index, $\beta_{LN}$ | Probability of Failure, p (f) |
|---|---|---|
| 0.65–1.66 | 0.1358 | 44.60 % |

# SLOPE CUT PROGRAM: NORMAL DISTRIBUTION DATA

*Crack in Upper Slope Face (Normal Distribution)*

| H | θ | b | Ω | $\gamma_w$ |
|---|---|---|---|---|
| 28 | 60 | 10 | 0 | 9.81 |

| | Value | Mean | σ | C.O.V. | $x_+$ | $x_-$ | ζ | λ | $x_+$ | $x_-$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Angle of Failure plane, $\psi_p$ | 20 | 20 | 3 | 0.15 | 23 | 17 | 0.1492 | 2.9846 | 3.1338 | 2.8354 |
| Cohesion, c | 100 | 100 | 15 | 0.15 | 115 | 85 | 0.1492 | 4.594 | 4.7432 | 4.4449 |
| Internal Friction Angle, ϕ | 20 | 20 | 3 | 0.15 | 23 | 17 | 0.1492 | 2.9846 | 3.1338 | 2.8354 |
| Rock Unit Weight, γ | 25.1 | 25.1 | 3.765 | 0.15 | 28.865 | 21.335 | 0.1492 | 3.2117 | 3.3609 | 3.0626 |
| $Z/Z_w$ | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |




| Result | Model I |
|---|---|

| F.S. | C.O.V. | $\sigma_F$ | $\beta_N$ | $p_f$ |
|---|---|---|---|---|
| 1.02 | 15% | 0.1526 | 0.1136 | 45.48% |

| Result | Model II |
|---|---|

| Result | | $F1_+$ | $F1_-$ | $F2_+$ | $F2_-$ | $F3_+$ | $F3_-$ | $F4_+$ | $F4_-$ |
|---|---|---|---|---|---|---|---|---|---|
| V | 1674.4604 | 1399.8021 | 1962.0608 | 1674.4604 | 1674.4604 | 1674.4604 | 1674.4604 | 1674.4604 | 1674.4604 |
| U | 2523.5114 | 2355.3795 | 2684.1919 | 2523.5114 | 2523.5114 | 2523.5114 | 2523.5114 | 2523.5114 | 2523.5114 |
| W | 9581.3048 | 9061.4281 | 10081.7194 | 9581.3048 | 9581.3048 | 9581.3048 | 9581.3048 | 11018.5005 | 8144.1090 |
| A | 27.8451 | 28.4255 | 27.3614 | 27.8451 | 27.8451 | 27.8451 | 27.8451 | 27.8451 | 27.8451 |
| Z | 18.4764 | 16.8933 | 20.0003 | 18.4764 | 18.4764 | 18.4764 | 18.4764 | 18.4764 | 18.4764 |
| Zw | 18.4764 | 16.8933 | 20.0003 | 18.4764 | 18.4764 | 18.4764 | 18.4764 | 18.4764 | 18.4764 |
| R | 4934.5778 | 4822.0974 | 5059.4875 | 5352.2538 | 4516.9017 | 5291.9947 | 4590.5410 | 5426.1277 | 4443.0279 |
| Q | 4850.4774 | 4829.1067 | 4823.9376 | 4850.4774 | 4850.4774 | 4850.4774 | 4850.4774 | 5342.0272 | 4358.9275 |
| F.S. | 1.02 | 1.00 | 1.05 | 1.10 | 0.93 | 1.09 | 0.95 | 1.02 | 1.02 |
| | | ΔF1/2 | (0.0251) | ΔF2/2 | 0.0861 | ΔF3/2 | 0.0723 | ΔF4/2 | (0.0018) |
| | | $\Delta F1^2$ | 0.0006 | $\Delta F2^2$ | 0.0074 | $\Delta F3^2$ | 0.0052 | $\Delta F4^2$ | 0.0000 |

| $F.S._{MLV}$ | $\sigma_F$ | $C.O.V._F$ | $\beta_N$ | R | P.F. |
|---|---|---|---|---|---|
| 1.02 | 0.1152 | 11.30% | 0.1676 | 0.5666 | 43.34% |

| Result | Model III |
|---|---|

| no. of iteration | 1000 |
|---|---|

RUN PROGRAM

| F.S. | P.F. | β | F.S. min | F.S. max |
|---|---|---|---|---|
| 1.02 | 42.70% | -0.1840 | 0.66 | 1.45 |

# SOURCE CODE OF SIMULATION: NORMAL DISTRIBUTION DATA

```
Function log_inv(m, sd)
    Randomize
    r = Rnd
    If sd <> 0 Then
        x = Application.WorksheetFunction.LogInv(r, m, sd)
        log_inv = x

    Else
        log_inv = Exp(m)
    End If
End Function
Sub find_PF_plan_norm1()
Dim k1, k2 As Double
Dim fs_min, fs_max As Double
Dim no_loop As Integer
Dim cal_FS As Double
    k1 = 0
    k2 = 0
    fs_min = Cells(56, 1)
    fs_max = Cells(56, 1)
    no_loop = Cells(53, 3)
    For i = 1 To no_loop
        For j = 0 To 4
            If j = 4 Then
                Cells(7 + j, 3) = norm_inv(Cells(7 + j, 4), Cells(7 +
j, 5))
                While (Cells(7 + j, 3) < 0) Or (Cells(7 + j, 3) > 1)
                   Cells(7 + j, 3) = norm_inv(Cells(7 + j, 4), Cells(7
+ j, 5))
                Wend
            Else
                Cells(7 + j, 3) = norm_inv(Cells(7 + j, 4), Cells(7 +
j, 5))
            End If
```

## SOURCE CODE OF SIMULATION: NORMAL DISTRIBUTION DATA (CONTINUE)

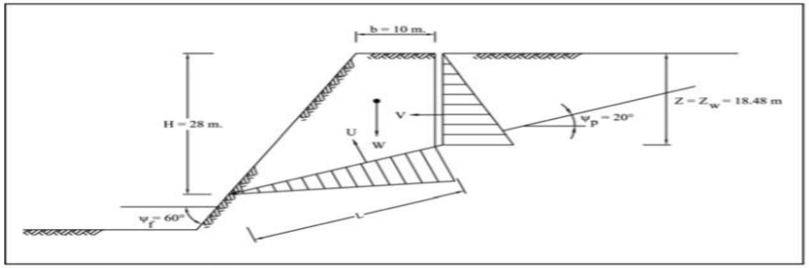
```
Next j
        Application.Calculate
        cal_FS = Cells(56, 1)
        If cal_FS < 1 Then
            k1 = k1 + 1
        Else
            k2 = k2 + 1
        End If
        Cells(56, 2) = k1 / (k1 + k2)
        Cells(53, 3) = no_loop - i
        If cal_FS < fs_min Then
            fs_min = cal_FS
            Cells(56, 4) = fs_min
        End If
        If cal_FS > fs_max Then
            fs_max = cal_FS
            Cells(56, 5) = fs_max
        End If
Next i
    For j = 0 To 4
        Cells(7 + j, 3) = Cells(7 + j, 4)
    Next j
    Cells(53, 3) = no_loop
End Sub
```

# SLOPE CUT PROGRAM: LOGNORMAL DISTRIBUTION DATA

***Crack in Upper Slope Face (Lognormal Distribution)***

| H | θ | b | Ω | $\gamma_w$ |
|---|---|---|---|---|
| 28 | 60 | 10 | 0 | 9.81 |

| | Value | Mean | σ | C.O.V. | $x_+$ | $x_-$ | ζ | λ | $x_+$ | $x_-$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Angle of Failure plane, $\psi_p$ | 20 | 20 | 3 | 0.15 | 23 | 17 | 0.1492 | 2.9846 | 3.1338 | 2.8354 |
| Cohesion, c | 100 | 100 | 15 | 0.15 | 115 | 85 | 0.1492 | 4.594 | 4.7432 | 4.4449 |
| Internal Friction Angle, ϕ | 20 | 20 | 3 | 0.15 | 23 | 17 | 0.1492 | 2.9846 | 3.1338 | 2.8354 |
| Rock Unit Weight, γ | 25.1 | 25.1 | 3.765 | 0.15 | 28.865 | 21.335 | 0.1492 | 3.2117 | 3.3609 | 3.0626 |
| $Z/Z_w$ | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |




| Result | Model I |
|---|---|

| F.S. | C.O.V. | $\sigma_F$ | $\beta_N$ | $p_f$ |
|---|---|---|---|---|
| 1.01 | 15% | 0.1517 | -0.0007 | 50.03% |

| Result | Model II |
|---|---|

| Result | | $F1_+$ | $F1_-$ | $F2_+$ | $F2_-$ | $F3_+$ | $F3_-$ | $F4_+$ | $F4_-$ |
|---|---|---|---|---|---|---|---|---|---|
| V | 1695.2377 | 1403.3355 | 1958.3479 | 1695.2377 | 1695.2377 | 1695.2377 | 1695.2377 | 1695.2377 | 1695.2377 |
| U | 2535.5743 | 2357.6602 | 2682.1941 | 2535.5743 | 2535.5743 | 2535.5743 | 2535.5743 | 2535.5743 | 2535.5743 |
| W | 9512.4119 | 8968.0953 | 9964.0306 | 9512.4119 | 9512.4119 | 9512.4119 | 9512.4119 | 11042.6367 | 8194.2368 |
| A | 27.8062 | 28.4172 | 27.3669 | 27.8062 | 27.8062 | 27.8062 | 27.8062 | 27.8062 | 27.8062 |
| Z | 18.5907 | 16.9146 | 19.9814 | 18.5907 | 18.5907 | 18.5907 | 18.5907 | 18.5907 | 18.5907 |
| Zw | 18.5907 | 16.9146 | 19.9814 | 18.5907 | 18.5907 | 18.5907 | 18.5907 | 18.5907 | 18.5907 |
| R | 4850.6615 | 4735.0508 | 4961.3825 | 5293.0201 | 4469.6024 | 5224.8876 | 4540.1666 | 5368.4720 | 4404.6061 |
| Q | 4814.1211 | 4790.5707 | 4791.9021 | 4814.1211 | 4814.1211 | 4814.1211 | 4814.1211 | 5331.9316 | 4368.0657 |
| F.S. | 1.01 | 0.99 | 1.04 | 1.10 | 0.93 | 1.09 | 0.94 | 1.01 | 1.01 |
| | | $\Delta F1/2$ | (0.0235) | $\Delta F2/2$ | 0.0855 | $\Delta F3/2$ | 0.0711 | $\Delta F4/2$ | (0.0008) |
| | | $\Delta F1^2$ | 0.0006 | $\Delta F2^2$ | 0.0073 | $\Delta F3^2$ | 0.0051 | $\Delta F4^2$ | 0.0000 |

| $F.S._{MLV}$ | $\sigma_F$ | $C.O.V._F$ | $\beta_{LN}$ | R | P.F. |
|---|---|---|---|---|---|
| 1.01 | 0.1137 | 11.23% | 0.0498 | 0.5199 | 48.01% |

| Result | Model III |
|---|---|

| no. of iteration | 1000 |
|---|---|

RUN PROGRAM

| F.S. | P.F. | β | F.S. min | F.S. max |
|---|---|---|---|---|
| 1.01 | 44.60% | -0.1358 | 0.65 | 1.66 |

## SOURCE CODE OF SIMULATION: LOGNORMAL DISTRIBUTION DATA

```
Function log_inv(m, sd)
    Randomize
    r = Rnd
    If sd <> 0 Then
        x = Application.WorksheetFunction.LogInv(r, m, sd)
        log_inv = x
    Else
        log_inv = m
    End If
End Function
Sub find_PF_plan_log1()
Dim k1, k2 As Double
Dim fs_min, fs_max As Double
Dim no_loop As Integer
Dim cal_FS As Double
    k1 = 0
    k2 = 0
    fs_min = Cells(56, 1)
    fs_max = Cells(56, 1)
    no_loop = Cells(53, 3)
    For i = 1 To no_loop
        For j = 0 To 4
            If j = 4 Then
                Cells(7 + j, 3) = log_inv(Cells(7 + j, 10),
Cells(7 + j, 9))
                While (Cells(7 + j, 3) < 0) Or (Cells(7 + j, 3)
> 1)
                    Cells(7 + j, 3) = log_inv(Cells(7 + j, 10),
Cells(7 + j, 9))
                Wend
            Else
                Cells(7 + j, 3) = log_inv(Cells(7 + j, 10),
Cells(7 + j, 9))
            End If
```

# SOURCE CODE OF SIMULATION: LOGNORMAL DISTRIBUTION DATA (CONTINUE)

```
Next j
        Application.Calculate
        cal_FS = Cells(56, 1)
        If cal_FS < 1 Then
            k1 = k1 + 1
        Else
            k2 = k2 + 1
        End If
        Cells(56, 2) = k1 / (k1 + k2)
        Cells(53, 3) = no_loop - i
        If cal_FS < fs_min Then
            fs_min = cal_FS
            Cells(56, 4) = fs_min
        End If
        If cal_FS > fs_max Then
            fs_max = cal_FS
            Cells(56, 5) = fs_max
        End If
    Next i
    For j = 0 To 4
        Cells(7 + j, 3) = Cells(7 + j, 4)
    Next j
    Cells(53, 3) = no_loop
End Sub
```

# CONCLUSION ON STABILITY PLANNING

- **Deterministic (or Conventional) Method**
- **Probabilistic Method Using Risk Models**
- **Improved Method – Better Decision – Efficient Operation**

MAHIDOL UNIVERSITY

*Wisdom of the Land*

**THANK YOU FOR YOUR ATTENTION**